Kwantumfysica I

2008-2009

Hoorcollege vrijdag 19 december 2008

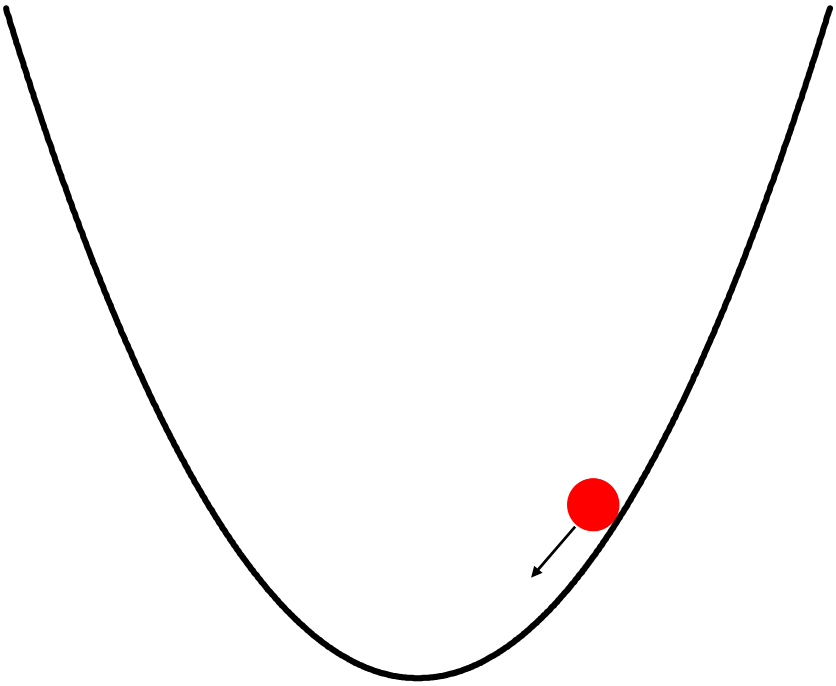
Deze week vooral Hoofdstuk 7 (beetje 8)

Vragen n.a.v. stof tot hier?

Vandaag

Harmonic oscillators, photons

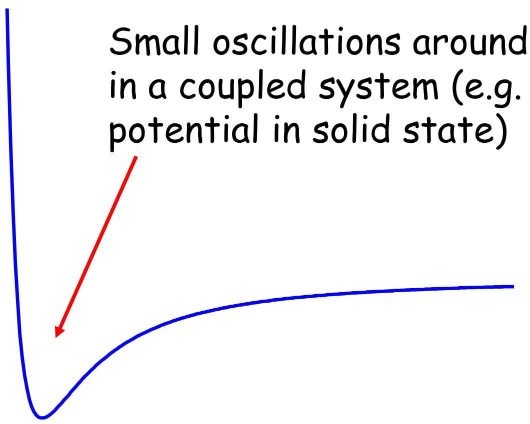
1D Harmonic oscillator



1D Harmonic oscillator
Very important model systems

EM waves (photons)

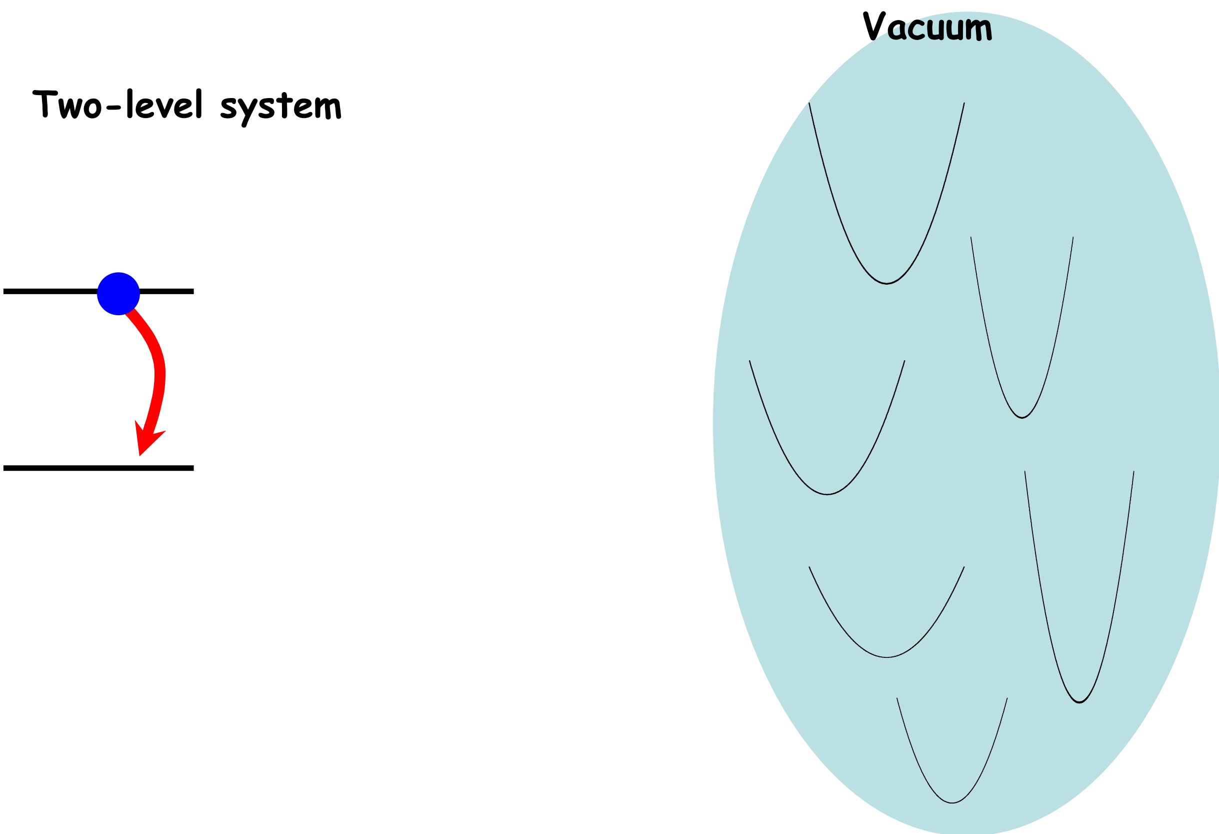
Lattice vibrations (phonons)



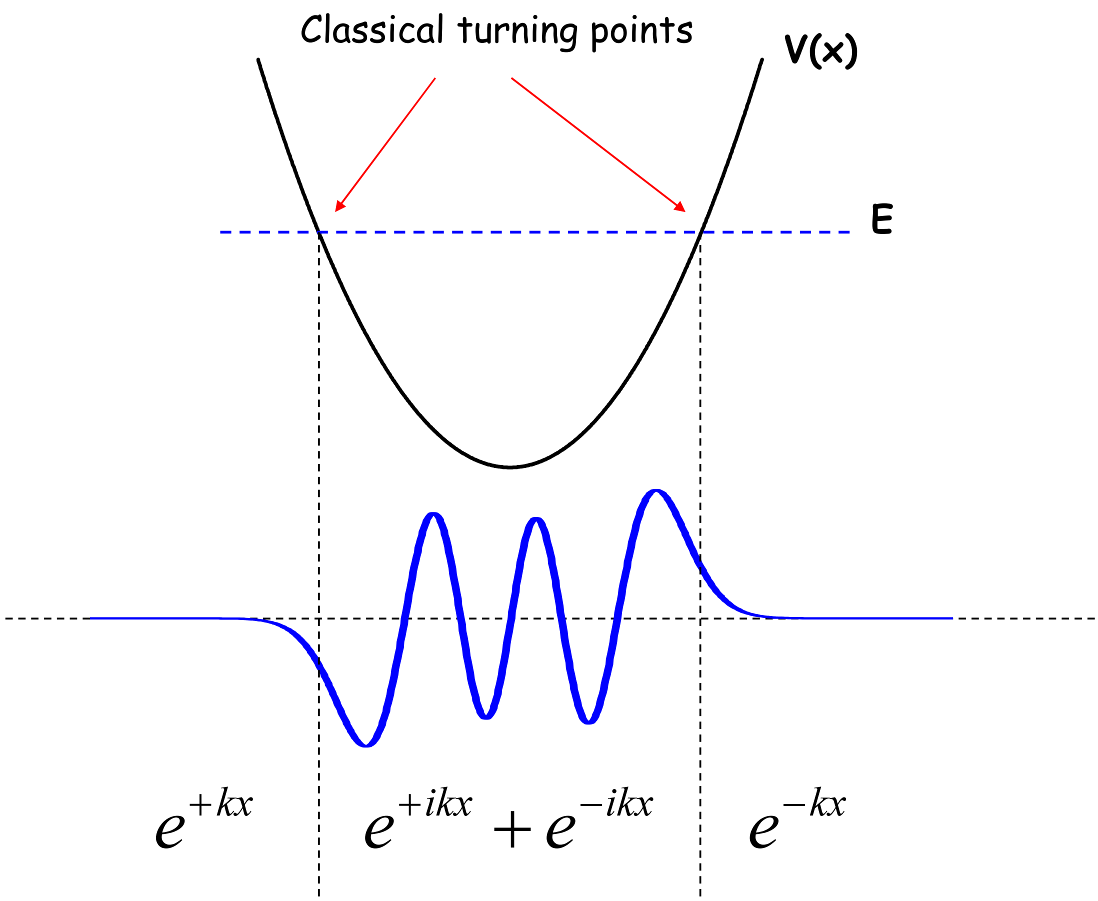
Small oscillations around equilibrium in a coupled system (e.g. effective potential in solid state)

$$V(x) = \frac{a}{x^6} - \frac{4a}{x^2}$$

Vacuum and spontaneous emission by 2-level systems



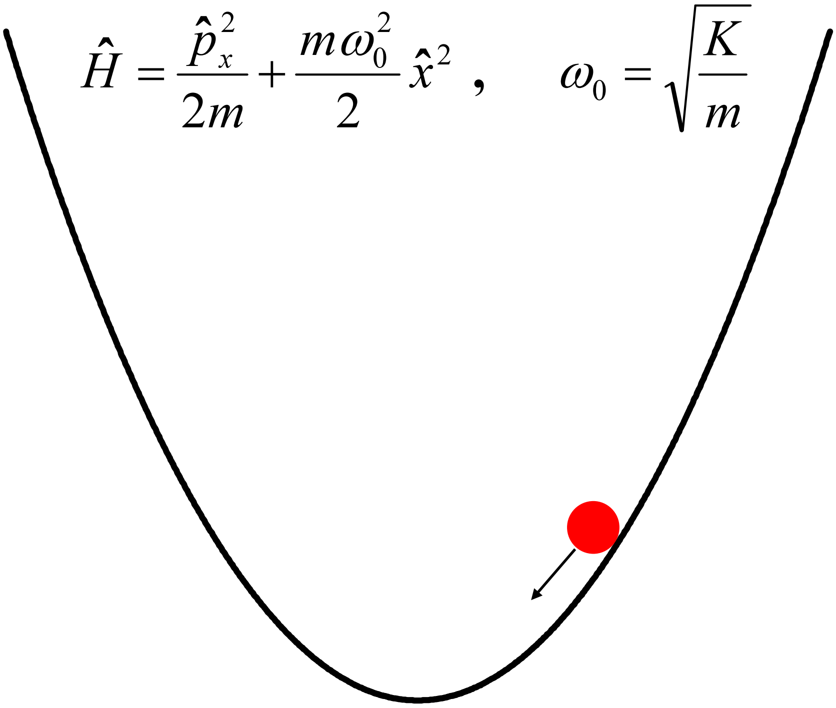
Solving time-independent Schrodinger equation – As in previous lecture



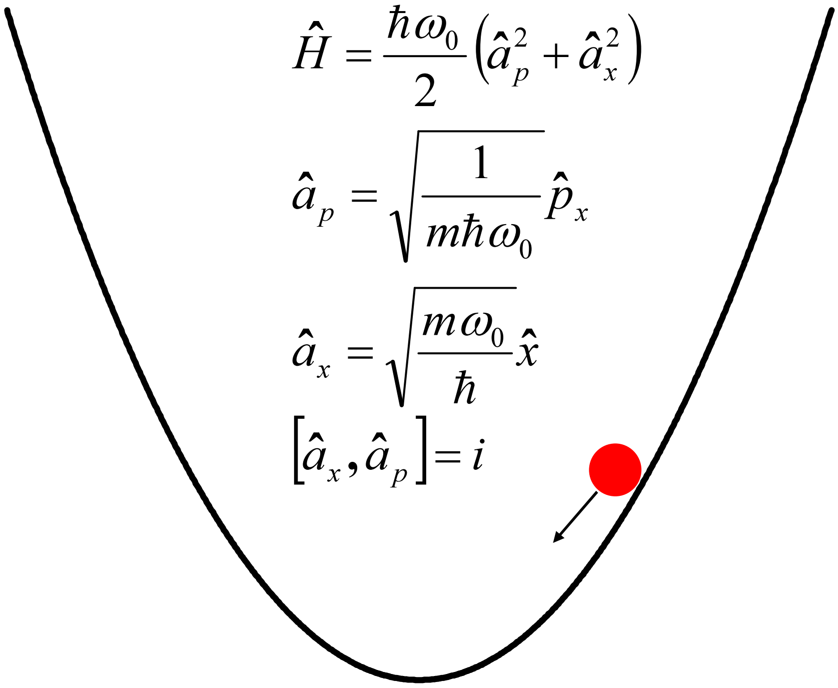
1D Harmonic oscillator

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{K}{2}\hat{x}^2$$

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{m\omega_0^2}{2}\hat{x}^2 \,, \qquad \omega_0 = \sqrt{\frac{K}{m}}$$



1D Harmonic oscillator



$$\hat{H} = \frac{\hbar\omega_0}{2}\left(\hat{a}_p^2 + \hat{a}_x^2\right)$$

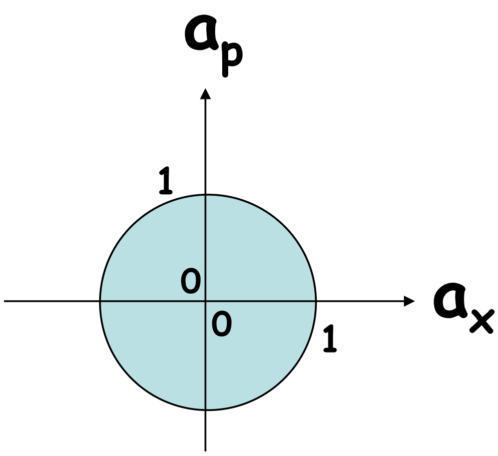
$$\hat{a}_p = \sqrt{\frac{1}{m\hbar\omega_0}}\,\hat{p}_x$$

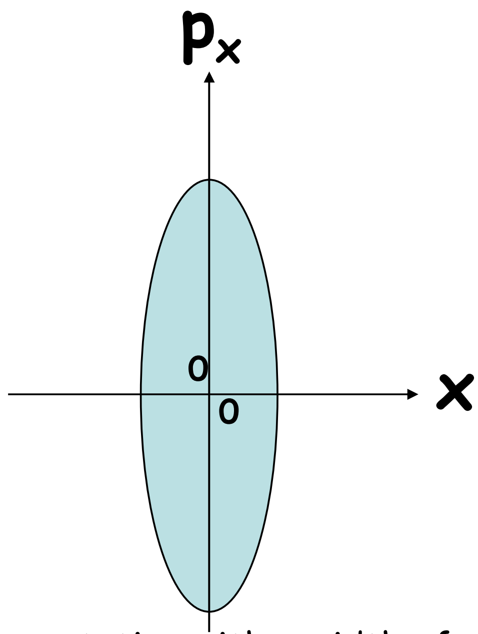
$$\hat{a}_x = \sqrt{\frac{m\omega_0}{\hbar}}\,\hat{x}$$

$$\left[\hat{a}_x, \hat{a}_p\right] = i$$

Any harmonic oscillator can be expressed in a pair of normalized, dimensionless, conjugate coordinates

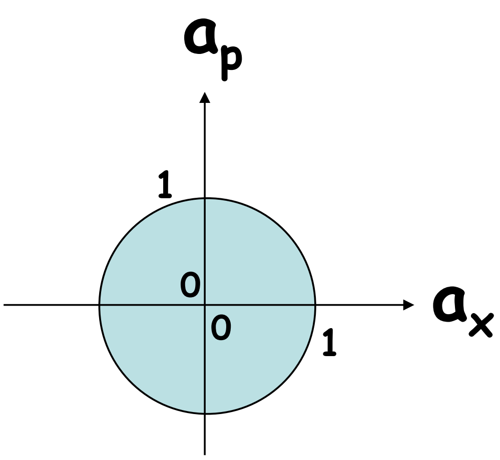
Any harmonic oscillator can be expressed in a pair of
<u>normalized</u>, <u>dimensionless</u>, <u>conjugate</u> coordinates.

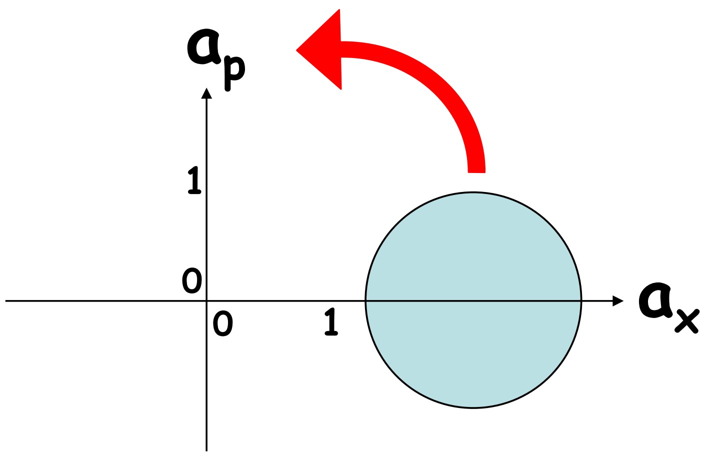




Gaussian wavepacket in a_x- and a_p-representation with a width of about 1 for both dimensionless coordinates. This can be represented as a circle in the a_x-a_p plane. In the plane of physical coordinates x and p_x, it is not a circle, but an ellipse with a shape that depends on m and K.

"Coherent state" – state that corresponds to classical oscillation



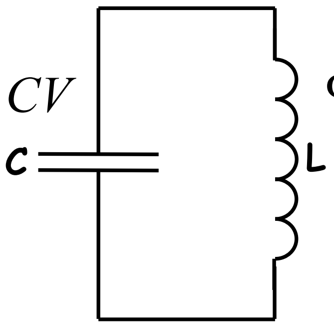


$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Gaussian wavepacket in a_x- and a_p-representation, that moves around in the a_x-a_p plane without changing shape.
The parameter α is proportional to the classical oscillation amplitude.

Other example of harmonic oscillator system that can be mapped on dimensionless coordinates a_x and a_p: LC circuit

$$Q = CV$$



$$\Phi = LI = L\frac{dQ}{dt}$$

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}, \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

$\frac{\hat{Q}^2}{2C}$ kinetic-energy-like term (C is "mass", Q is "momentum")

$\frac{\hat{\Phi}^2}{2L}$ potential-energy-like term (Φ is "position")

$$\hat{H} = \frac{\hbar\omega_0}{2}\left(\hat{a}_p^2 + \hat{a}_x^2\right)$$

$$\hat{a}_p = \sqrt{\frac{1}{C\hbar\omega_0}}\hat{Q}$$

$$\hat{a}_x = \sqrt{\frac{C\omega_0}{\hbar}}\hat{\Phi}$$

$$\left[\hat{a}_x, \hat{a}_p\right] = i$$

$$\begin{cases} \hat{a} = \frac{1}{\sqrt{2}}\left(\hat{a}_x + i\hat{a}_p\right) \\ \hat{a}^+ = \frac{1}{\sqrt{2}}\left(\hat{a}_x - i\hat{a}_p\right) \end{cases}$$

Annihilation/destruction operator

Creation operator

Non-Hermitian operators!

$$\begin{cases} \hat{a}_x = \frac{1}{\sqrt{2}}\left(\hat{a}^+ + \hat{a}\right) \\ \hat{a}_p = \frac{i}{\sqrt{2}}\left(\hat{a}^+ - \hat{a}\right) \end{cases}$$

$$\left[\hat{a}, \hat{a}^+\right] = 1$$

Why use this notation?

Algebraic convenience

Physical meaning op creation and annihilation

$$\left[\hat{a},\hat{a}^{+}\right]=1$$

$$\hat{a}\hat{a}^{+}-\hat{a}^{+}\hat{a}=1$$

$$\hat{a}\hat{a}^{+}=\hat{a}^{+}\hat{a}+1$$

$$\hat{a}^{+}\hat{a}=\hat{N}$$

$$\hat{a}\hat{a}^{+}=\hat{N}+1$$

$$\hat{H}=\hbar\omega_0\left(\hat{N}+\frac{1}{2}\right)$$

$$\hat{H}\left|n\right\rangle=\hbar\omega_0\left(n+\frac{1}{2}\right)\left|n\right\rangle$$

Nature of eigensates leads to the concept of PHOTONS

$$\hat{N}\left|n\right\rangle=n\left|n\right\rangle$$

$\hat{a}\left|n\right\rangle=\sqrt{n}\left|n-1\right\rangle$ Annihilation/destruction operator

- removes a photon from a state

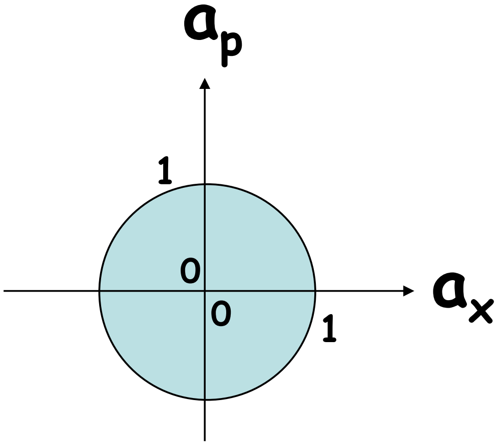
$\hat{a}^{+}\left|n\right\rangle=\sqrt{n+1}\left|n+1\right\rangle$ Creation operator

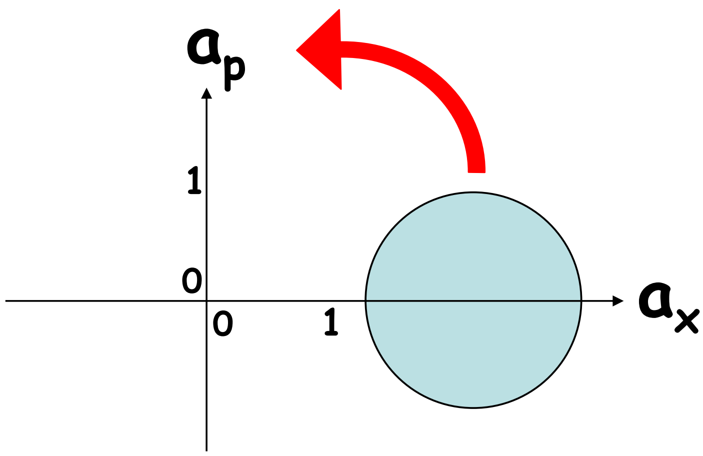
- adds a photon to a state

$$\boxed{\hat{a}\left|0\right\rangle=0}$$

Only positive photon numbers

"Coherent state" **– state that corresponds to classical oscillation**





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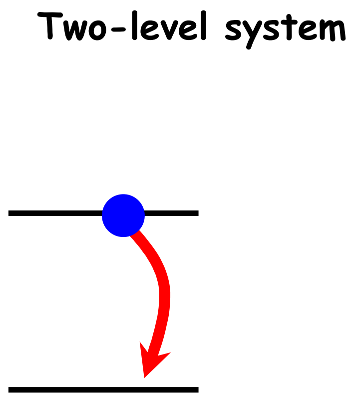
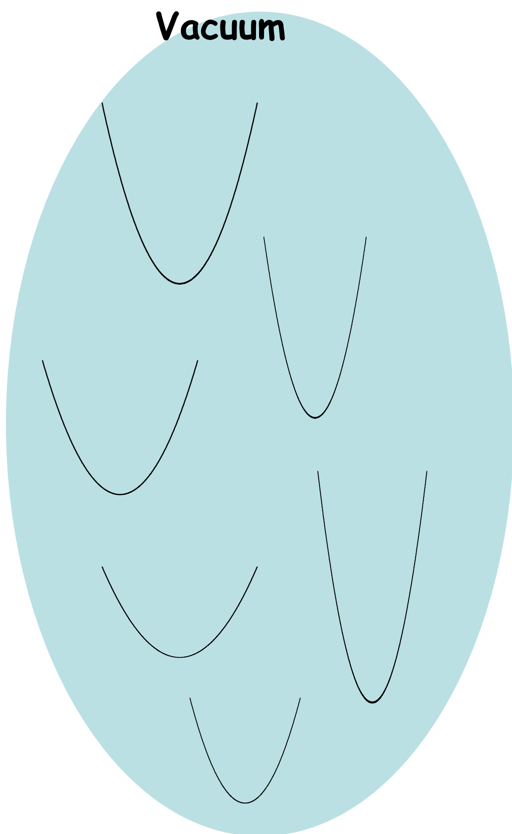
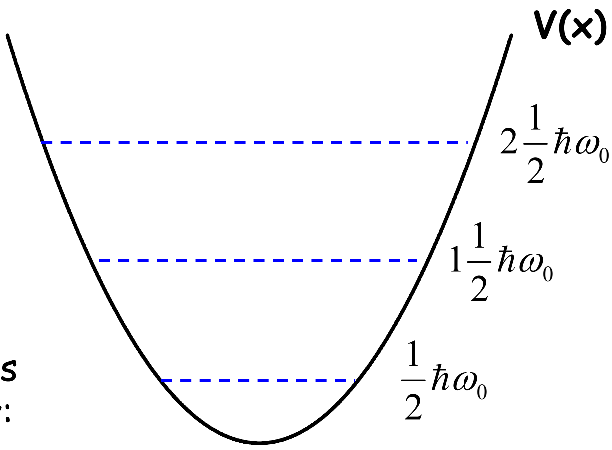
Vacuum and spontaneous emission by 2-level systems





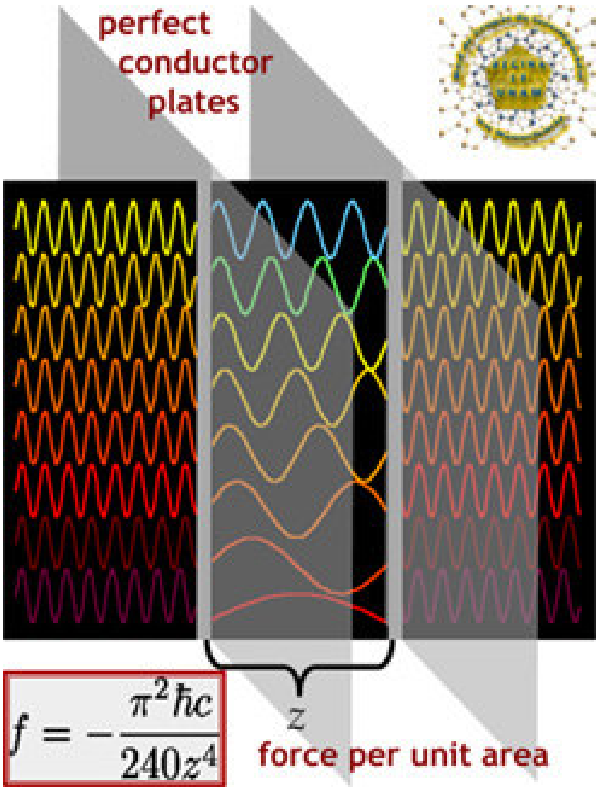
Zero-point or vacuum fluctuations

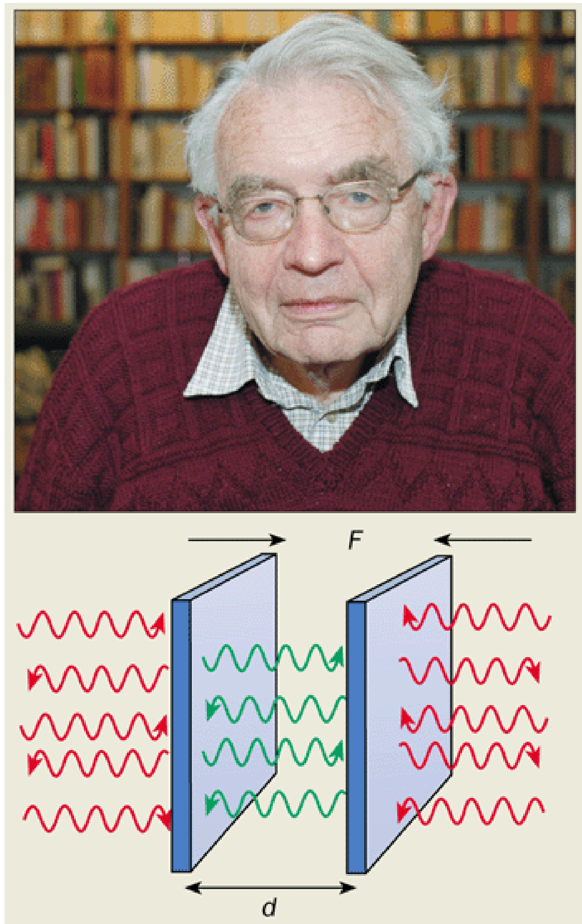


Groundstate has nonzero energy: Zero-point or vacuum fluctuations

$$\hat{H} = \hbar\omega_0\left(\hat{N} + \frac{1}{2}\right)$$

$$\hat{H}\left|n\right\rangle = \hbar\omega_0\left(n + \frac{1}{2}\right)\left|n\right\rangle$$


perfect
conductor
plates
$$f = -\frac{\pi^2 \hbar c}{240 z^4}$$
z
force per unit area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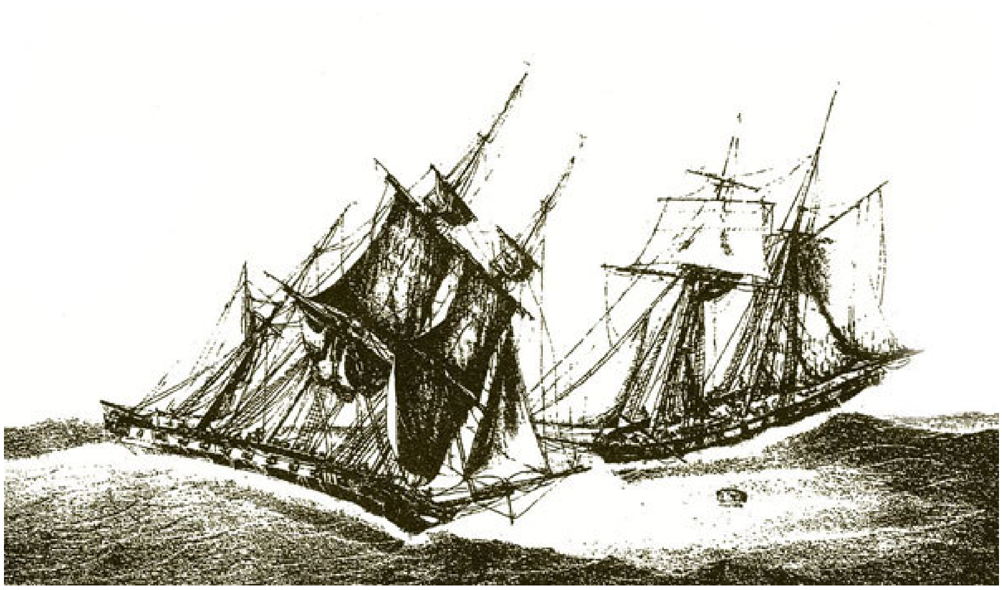

Samenvatting:

Harmonic oscillator, photons

Laatste colleges:

System with 2 particles
Coupling 2 quantum systems: LCAO
From 1D potential well to solid state
Angular momentum