Kwantumfysica I

2008-2009

Hoorcollege vrijdag 9 januari 2009

Deze week delen van Hoofdstuk 7 en 8

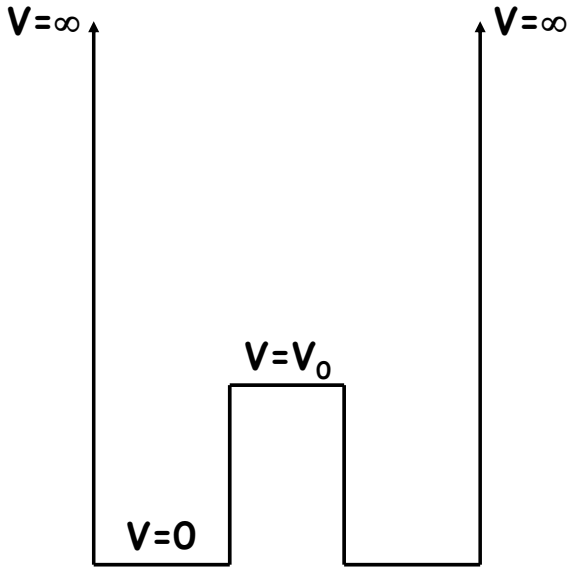
Vragen n.a.v. stof tot nu toe?

Vandaag

1. Weakly coupling 2 quantum systems
2. From particle in a well to solid state physics

Coupling 2 quantum systems: LCAO

Weakly couple two quantum wells, with a single particle in the total system.
What is the new wavefucntion for the ground state?



The book uses here a variational method (p. 331).
We will use here today another approach where we directly calculate the new energy eigenstates and eigenvalues of the system if the two wells are coupled, by solving the time-independent Schrödinger equation.

Two coupled wells

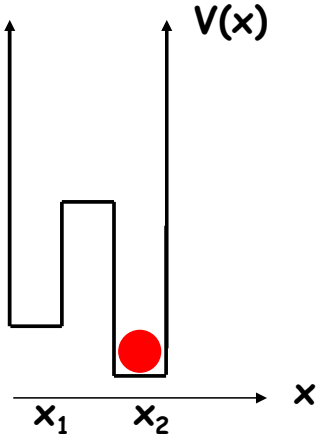
$$\hat{H} = \hat{H}_E + \hat{H}_T \leftrightarrow \begin{pmatrix} E_1 & T \\ T & E_2 \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} + \begin{pmatrix} 0 & T \\ T & 0 \end{pmatrix}$$

Tunnel coupling gives OFF-DIAGONAL ELEMENTS

E_i beschrijft de energie van het systeem in put bij x_i

T beschrijft de energie van het mechanisme dat overgangen van put 1 naar 2 en vice versa mogelijk maakt.

Mechanisch systeem, met 2 toestanden



Assume $E_2 = E_1$

$$\hat{H} = \hat{H}_E + \hat{H}_T \leftrightarrow \begin{pmatrix} E_1 & T \\ T & E_1 \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_1 \end{pmatrix} + \begin{pmatrix} 0 & T \\ T & 0 \end{pmatrix}$$

$$\begin{pmatrix} E_1 & T \\ T & E_1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}_i = E_i \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}_i \quad \Rightarrow$$

New eigenvalues E_g and E_e

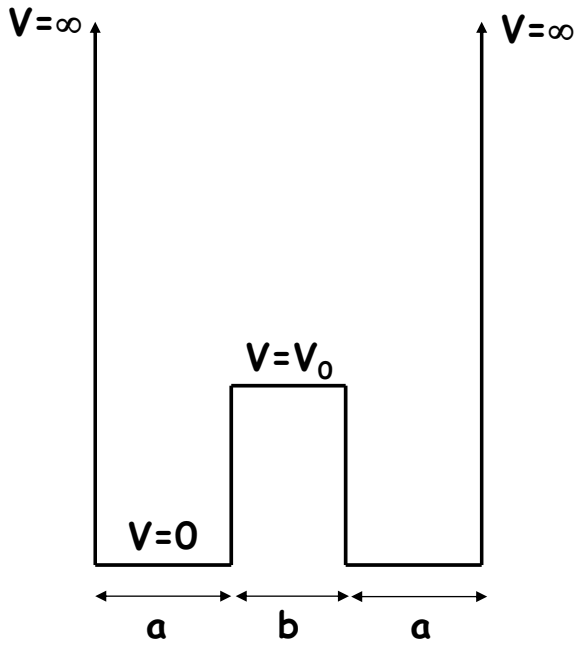
(Note that this equation is the time-indepent Schrödinger equation)

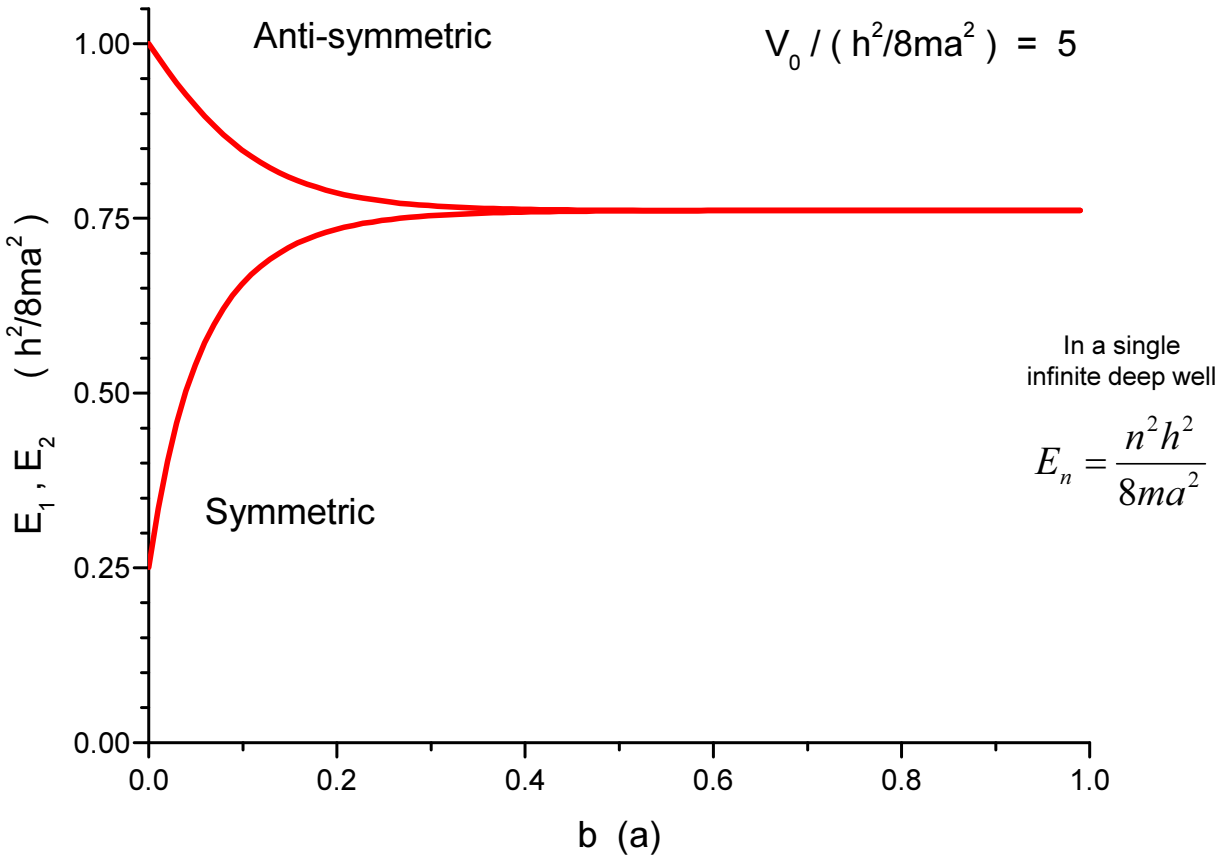
Fill in for the general solution for 2x2 eigenvalue problem:

$$\begin{cases} E_g = \dfrac{E_1 + E_1}{2} + \dfrac{1}{2}\sqrt{(E_1 - E_1)^2 + 4T^2} = E_1 + T \\ E_e = \dfrac{E_1 + E_1}{2} - \dfrac{1}{2}\sqrt{(E_1 - E_1)^2 + 4T^2} = E_1 - T \end{cases}$$

$$\left(\text{the choice of } + \text{ and } - \text{ here assumes } T < 0\right)$$

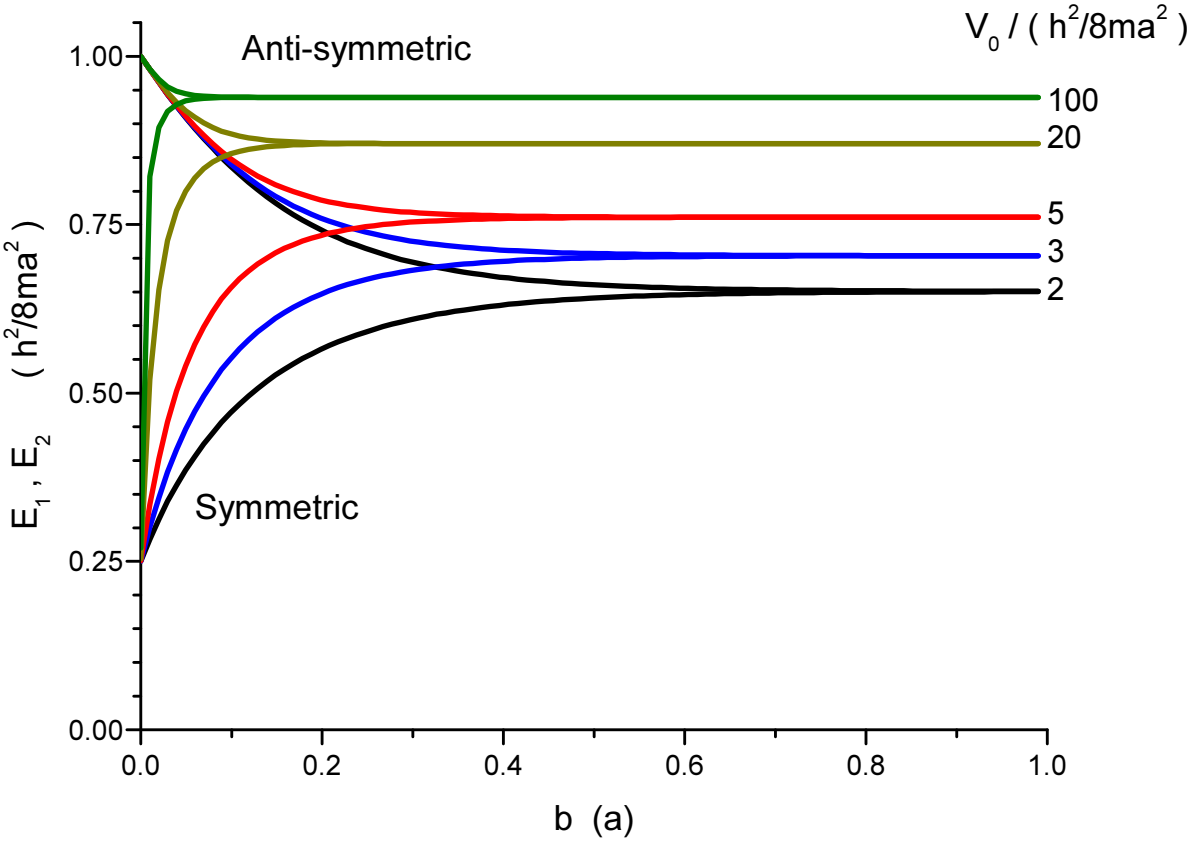
What is the wavefucntion for the ground state?

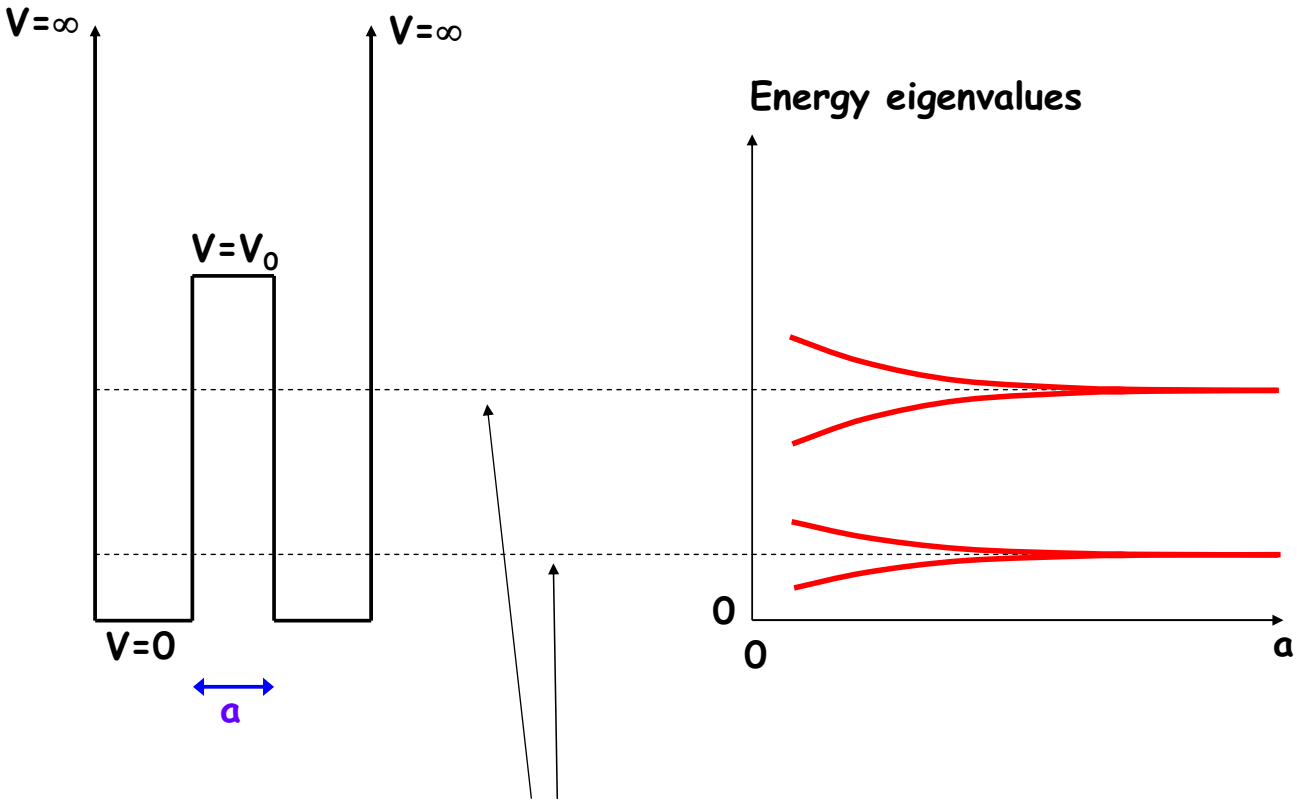




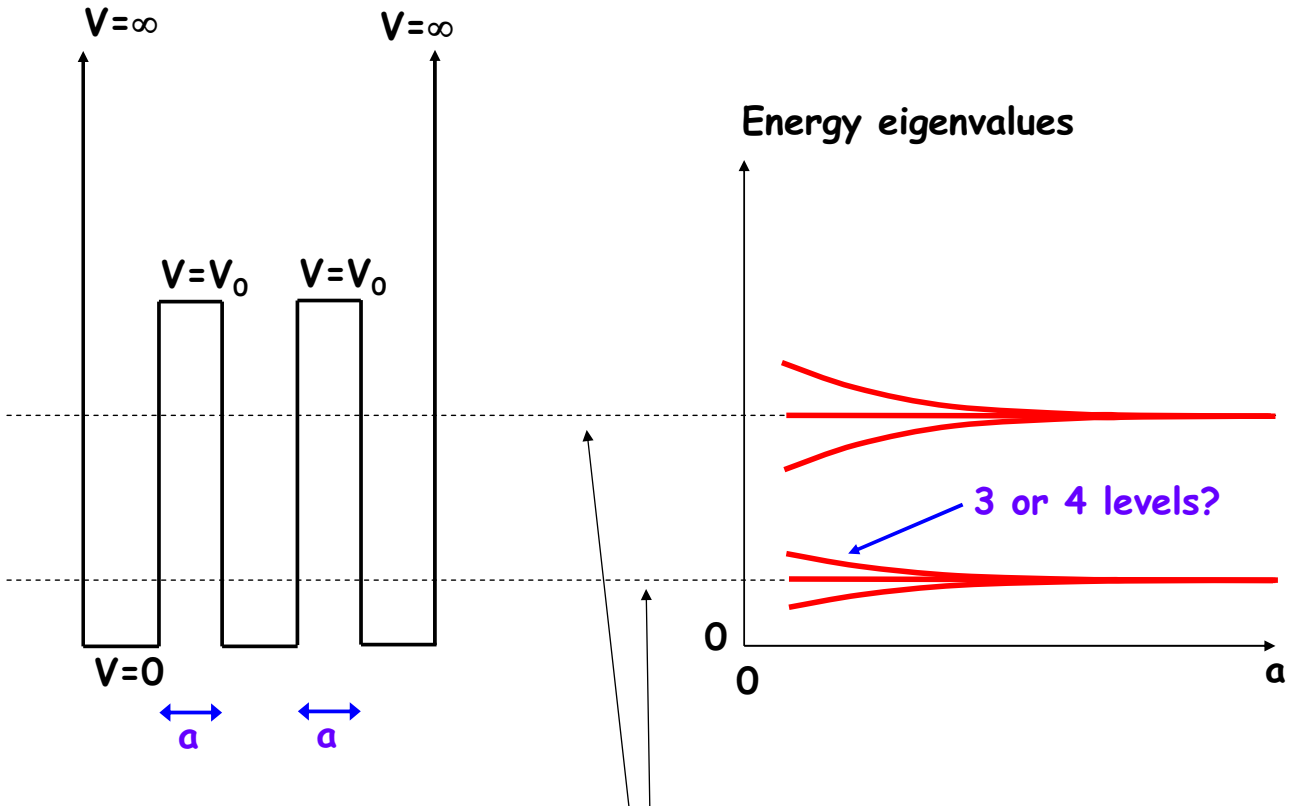
In a single infinite deep well

$$E_n = \frac{n^2 h^2}{8ma^2}$$

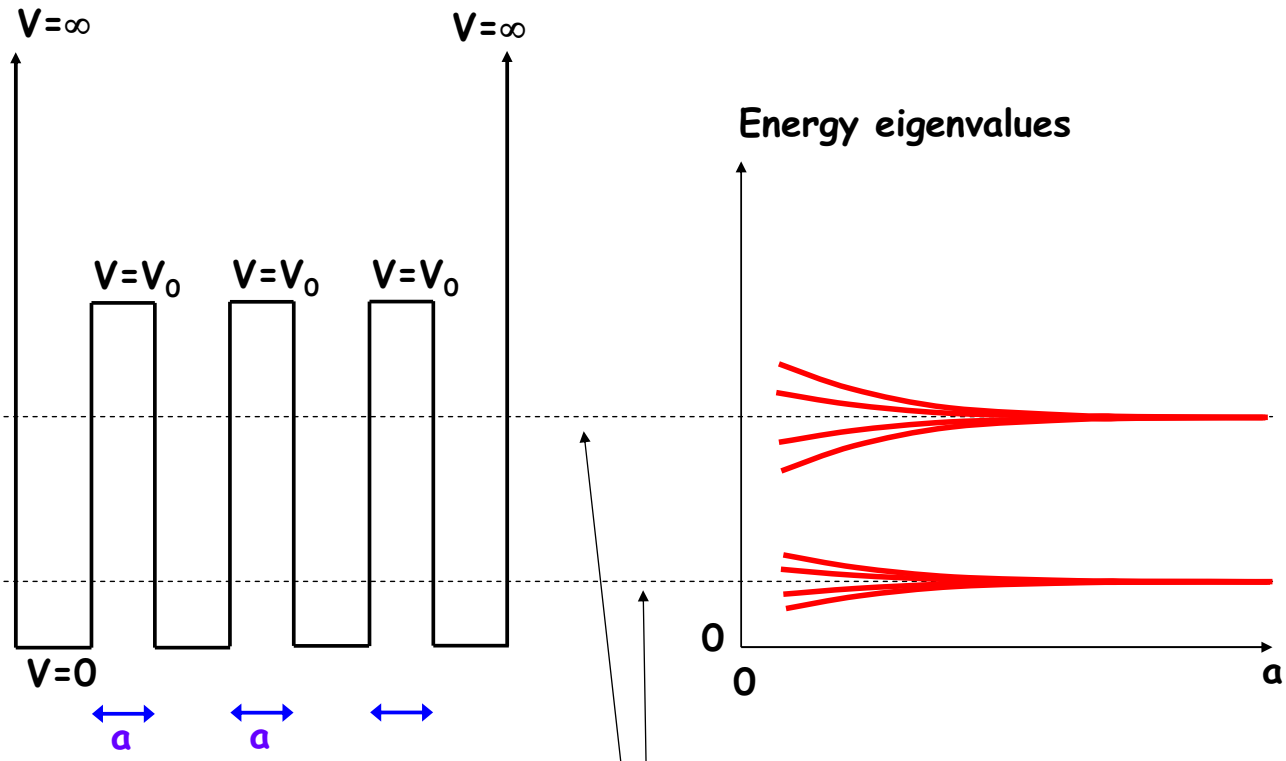




Energy levels for isolated potential wells



Energy levels for isolated potential wells



Energy levels for isolated potential wells

From 1D potential well to solid state material

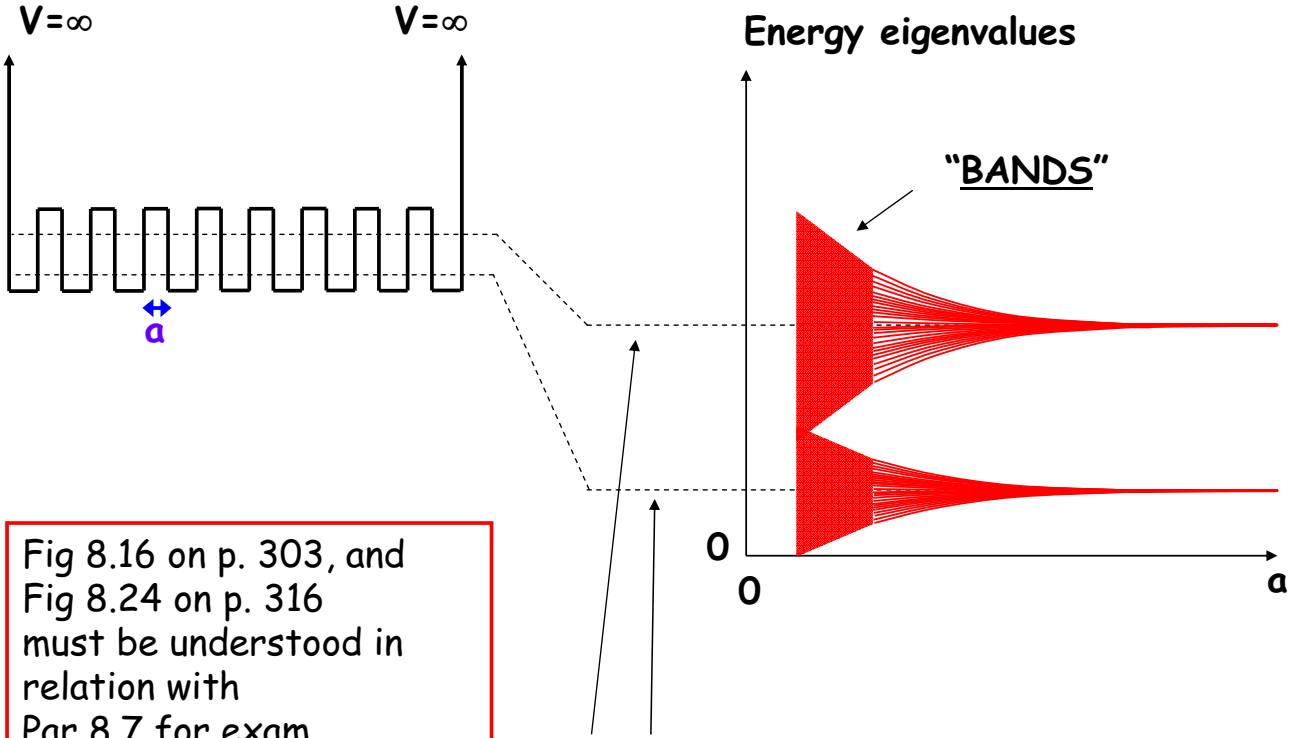


Fig 8.16 on p. 303, and
Fig 8.24 on p. 316
must be understood in
relation with
Par 8.7 for exam

Energy levels for isolated potential wells

Why is gold a conductor, and glass not?

(worked out on the blackboard)

Nog even stilstaan bij *LCAO* van vorig college,

en de eigenwaarden van 2x2 matrix

-teken van koppel matrix element T ?

-grootte van koppel matrix element en overlap?

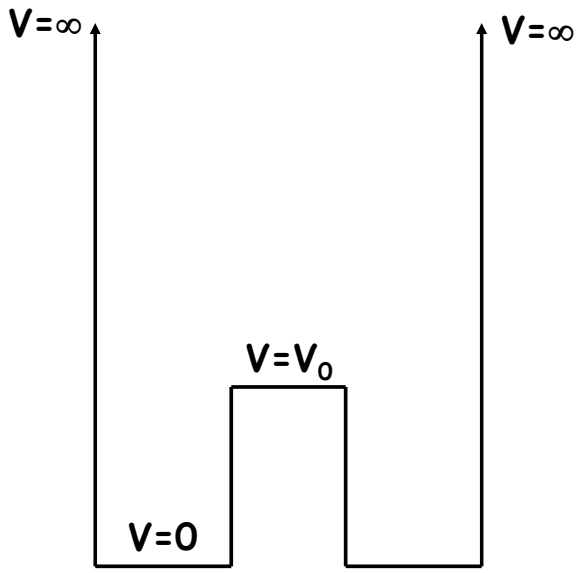
-is de parallel of anti-parallel toestand de grondtoestand?

(boek p. 334, en problem W7.1 op werkcollege)

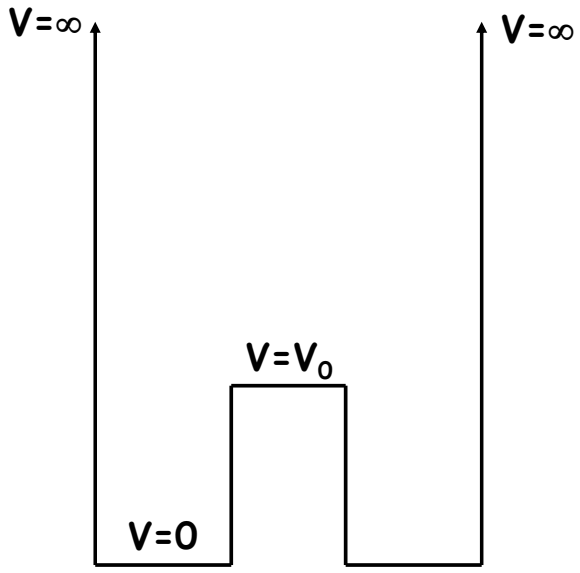
Probeer dit zelf uit door T reëel te nemen, en $T>0$ en $T<0$ de eigenstoestanden te onderzoeken, door simpelweg te kijken voor welke eigentoestanden en eigenwaarden de time-indepent Schrödinger equation op de vorige slide consistente oplossingen geeft. Gebruik ook wegens symmetrie voor het geval $E_2 = E_1$, dat $|c_1| = |c_2| = 1/\sqrt{2}$.

Complete uitwerking is op het bord gedaan.

What is the wavefucntion for the ground state?



How to form a state with the particle only on the left?



What is the dynamics for $t>0$
if the particle was only on the left at $t=0$?